

# VERIFICATION OF MATHEMATICAL MODEL FOR SMALL POWER SOURCES

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**Abstract:** This paper describes a method of verifying the mathematical model of the power supply. Simple equations describing the specific energy source. As an example photovoltaic panel was chosen. Simultaneously, there are described equation for calculating the solar radiation incident on the optionally oriented surface. These equations are used to model and convert the radiation which is recorded by meteorological stations. Reference panel and load are used for model validation. The goal of this paper is to describe a model of energy sources and its validation.

**Keywords:** Model of energy sources, solar cell, active load.

## NOMENCLATURE

- $I_0$  is dark saturation current in STC (A)
- $q$  is electron charge =  $1.6 \cdot 10^{-19} \text{ C}$
- $n$  is diode quality factor
- $k$  is Boltzman constant =  $1.3805 \cdot 10^{-23} \text{ JK}^{-1}$
- $T$  is temperature in Kelvin
- $I_L$  is light generated current of PV module (A)
- $R_S$  is series resistance
- $R_{SH}$  is parallel shunt
- $K_I$  is temperature coefficient of the short-circuit current
- $I_{SCS}$  is short-circuit current in STC
- $\theta_{ISC}$  is temperature coefficient (%/K)
- $\lambda$  is the PV module illumination ( $\text{W/m}^2$ )
- $\lambda_S$  is referral illumination =  $1000 \text{ W/m}^2$
- $V_{OCS}$  is open-circuit voltage in STC
- $K_V$  is temperature coefficient of the open-circuit voltage
- $I_{mpps}$  is current in maximum power point for STC
- $V_{mpps}$  is voltage in maximum power point for STC

## 1. INTRODUCTION

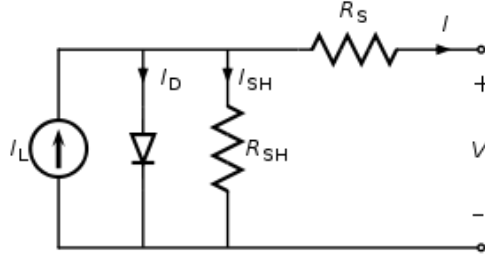
Feedback for the mathematical model is comparison with the real values of the monitored results and the simulations for the same physical conditions. The best result is for the largest possible amount of data in all operating conditions. The real conditions are unique because they are consisting of combination of large external effects and operating conditions. The situation can be simulated in laboratory conditions using climate chambers (temperature, humidity, pressure etc.), wind tunnel, artificial lighting etc. according to the specific requirements of individual power sources of energy such as small wind turbine, solar panels, fuel cell and others. At the same time, it is necessary to simulate the operating load of source on the whole range. For this purpose, an active load was designed. Primarily, it was designed to test the modules, but as already mentioned it can be advantageously used for other small sources of electricity. In the following text usage mainly for photovoltaic panel is discussed. Testing will be performed in real conditions on the reference model

with a parallel recording of the important parameters of external physical values with the use of weather station and necessary sensors.

## 2. MATHEMATICAL MODEL FOR A PHOTOVOLTAIC MODULE

### 2.1. BASIC SINGLE DIODE PV CELL MODEL

For the calculations and comparing is used simple one diode replacement scheme. This scheme is described by the following equations.



**Figure 1:** Circuit diagram of single diode PV cell model

Default Shockley equation describes the characteristics of semiconductor diodes.

$$I_D = I_0 \cdot (e^{\frac{q \cdot V}{n k T}} - 1) \quad (1)$$

After modifying substitute into the equation PV cell model. Which is expressed by using current.

$$I = I_L - I_D - I_{SH} \quad (2)$$

$$I = I_L - I_0 \left( e^{\frac{q \cdot (V + I \cdot R_S)}{n k T}} - 1 \right) - \frac{V + I \cdot R_S}{R_{SH}} \quad (3)$$

Other mathematical modifications can obtain equations for each coefficient describing the PV cell.

### 2.2. CONVERSION PARAMETERS OBTAINED UNDER STC

In the case that the module parameters are determined under defined test conditions, we can convert these values according to our chosen conditions (temperature, solar radiation). Values are listed on the label panel and in the datasheet, or you can get it as a result of laboratory tests.

Example table of parameters provided by the manufacturer of the panel is given in chapter reference model.

Conversion example of the temperature coefficient of the values specified percentage.

$$K_I = I_{SCS} \cdot \frac{\theta_{ISC}}{100} \quad (4)$$

Calculation of the short current of the values obtained for STC.

$$I_{SC} = I_{SCS} \cdot \frac{\lambda}{\lambda_S} + K_I \cdot (T - 298) \quad (5)$$

Calculation of the open voltage of the values obtained for STC.

$$V_{OC} = V_{OCS} + \frac{n k T}{q} \cdot \ln \left( \frac{\lambda}{\lambda_S} \right) + K_V (T - 298) \quad (6)$$

Calculation of the maximum power point current.

$$I_{mpp} = I_{mppS} \cdot \frac{\lambda}{\lambda_S} + K_I \cdot (T - 298) \quad (7)$$

Calculation of the maximum power point voltage.

$$V_{mpp} = V_{mppS} + \frac{nkT}{q} \cdot \ln\left(\frac{\lambda}{\lambda_S}\right) + K_V(T - 298) \quad (8)$$

To simulate and confirm the operating states of the PV module, data from meteorological stations can be used advantageously. Obtained information can be advantageously used about the properties of a particular device at different temperatures and exposure during the year. The network of meteorological stations in the Czech Republic and neighboring countries are very dense, so you can get relevant data for almost any location. Example of master data from the weather station are shown in Table 1.

Date	Global irradiation	Tracking Beam	Tmp	Wind Speed	Wet Bulb Temp	Wind direction	Cloud cover
200115 11:00:00	356	353	250	0	167	00	0
200115 11:01:00	381	357	272	14	166	12	0

**Table 1:** Example of data from weather station

Values of the weather station need to be further processed. It is necessary to adjust the basic unit according to the values of a particular record. They are usually the records for existent time irradiance on horizontal positioning pyranometer sensor, temperature, wind speed and direction, humidity, cloud cover and less frequently the intensity of light incident perpendicular to the pyranometer. This required much more complex system of comprising the measuring device and positioning of the pyranometer. PV modules are in the most cases placed statically so it is necessary to recalculate the value of radiation for each monitored stretch.

### 3. REFERENCE MODEL

For the reference model was used PV panel producer SOLARWATT blue 60p of nominal power 240Wp. Internal wiring panel consists of 60 solar cells connected in series.

Nominal power $P_N$	240WP
Nominal voltage $U_{mpp}$	29,9 V
Nominal current $I_{mpp}$	8,03 A
Opencircuit voltage $V_{OCS}$	37,2 V
Shortcircuit current $I_{SCS}$	8,5 A
$\Theta_{PN}$ Temperature Coefficient of $P_N$	-0,38%/K
$\Theta_{UOC}$ Temperature Coefficient of $V_{OC}$	-0,33%/K
$\Theta_{ISC}$ Temperature Coefficient of $I_{SC}$	0,04%/K

**Table 2:** Parameters of solar modul for STC

Decrease of the module efficiency with decrease of the irradiance from 1000 W/m<sup>2</sup> to 200 W/m<sup>2</sup> (25°C): 4<sup>±2</sup> % (relative) / -0,6<sup>±0,3</sup> % (absolute).

PV panels are consisting of multiple PV cell connected in series. Therefore, it is necessary to consider it while designing the model. In our case, it is only a series combination of photovoltaic cells. The equation considering this connection is shown below.  $N_s$  coefficient represents the number of series-connected cells in panel.

$$I_{PV} = I_L - I_0 \left( e^{\frac{q \cdot (V + I \cdot R_S)}{N_s \cdot nkT}} - 1 \right) - \frac{V + I \cdot R_S}{R_{SH}} \quad (9)$$

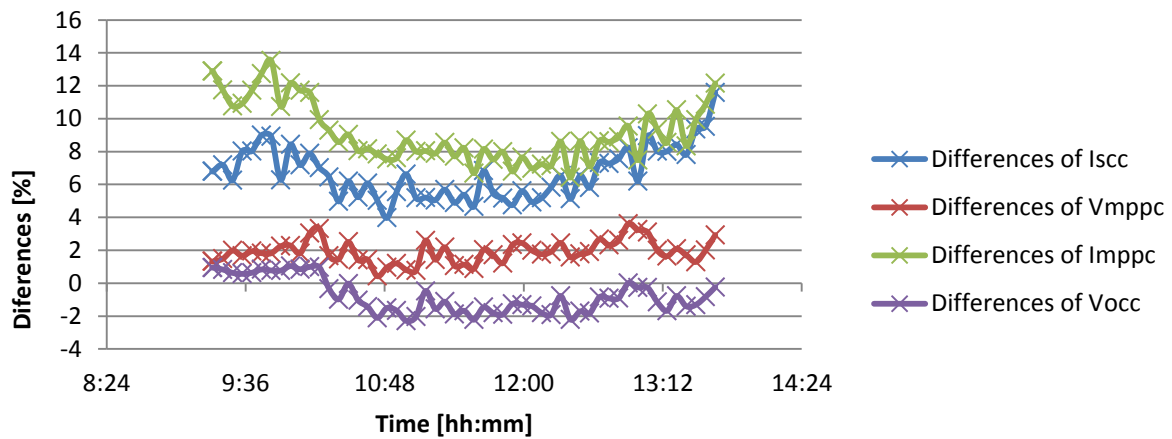
#### 4. CONFIRMATION OF MATHEMATICAL MODEL

To verify the mathematical model described previously the reference PV panel is used. The panel is positioned horizontally to the ground, so it is not necessary to recalculate the value of the irradiance measured by the pyranometer.

The weather conditions are recorded during the measurement using the weather station. The photovoltaic panel is connected to multichannel analyzer for recording the measured values. The output is loaded with a resistive load.

The graphs described waveforms with characteristic values of the panel measured with changing ambient conditions (temperature, solar radiation). Measured values were used to calculate the panel parameters by means of the above mathematical model. Additionally the calculated parameters include index letter "c".

By comparing the measured and calculated results was obtained from the following graph showing the difference of mathematical model results from measured values. Differences of all the parameters are significant, for current more than 10 %.



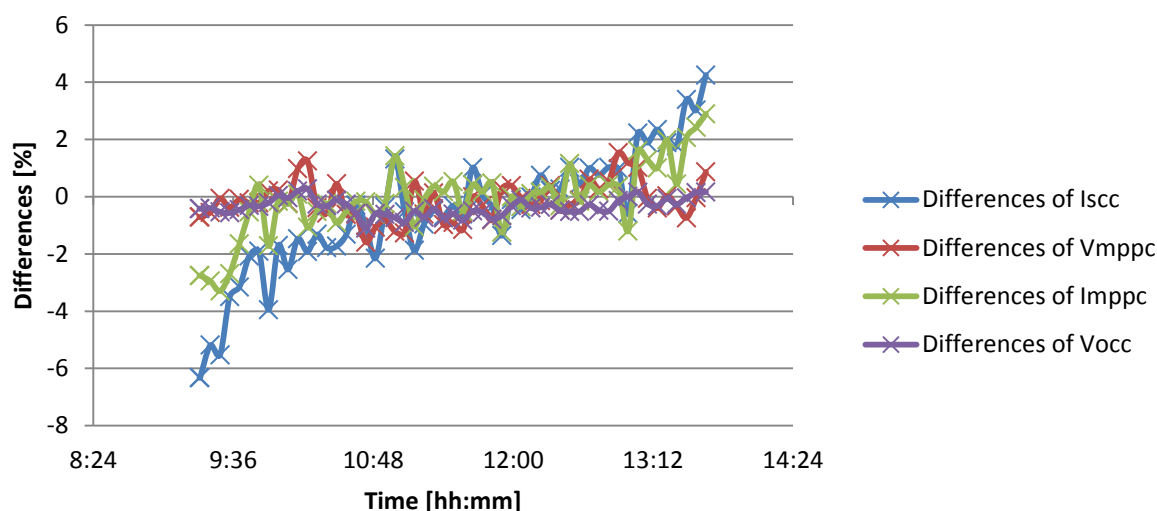
**Figure 2:** Differences between calculated and measured values

For the previous calculations were used parameters specified by the manufacturer of the PV panel at STC. Due to the degradation of the PV panel, contact resistance, resistance connecting cables etc. are the parameters measured PV panel were different than described in the catalog. From measured waveforms CV characteristics were determined parameters of the PV panel.

Nominal power $P_N$	225W <sub>p</sub>
Nominal voltage $U_{mpp}$	29,3 V
Nominal current $I_{mpp}$	7,7 A
Opencircuit voltage $V_{OCs}$	36,6 V
Shortcircuit current $I_{SCs}$	8,3 A
ΘPN Temperature Coefficient of $P_N$	-0,38%/K
ΘUOC Temperature Coefficient of $V_{OC}$	-0,1%/K
ΘISC Temperature Coefficient of $I_{SC}$	0,085%/K

**Table 3:** Recalculation panel parameters

We used the recalculated parameters again in the mathematical model. In comparison it is obvious that difference from the real data is much lower, for each parameters lower than 4 %.



**Figure 3:** Differences between calculated and measured values after recalculated

## 5. CONCLUSION

There was an example of a simple mathematical model of the PV panel and the procedure for its verification. Further work will deal with the production of the active load for the automation of measurement and continuous loading of small sources of power energy. Described model is suitable for calculating energy balances, for precise modeling is necessary to use more sophisticated replacement scheme. However the ways of verifying the model will be the same.

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